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Letter

Do black holes with generalized entropy violate Bekenstein bound?

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ABSTRACT

In general yes, but also not quite. It is known that if the Bekenstein-Hawking entropy is replaced by some kind of generalized entropy, then the Bekenstein bound may be grossly violated. In this work, we show that this undesired violation can be avoided if we employ the equivalence between generalized entropy and varying-G gravity (GEVAG). In this approach, modifying entropy necessarily also modifies gravity (as one should expect if gravity is indeed inherently tied to thermodynamics), which leads to an effective gravitational "constant" G_{eff} that is area-dependent, and a thermodynamic energy that is distinct from the ADM mass. We show that a relaxed Bekenstein bound of the form $S \leqslant CRE$ is always satisfied, albeit the coefficient C is no longer 2π.

1. Introduction: generalized entropy and Bekenstein bound

A system with entropy S, a suitably defined size R and an energy E satisfies the standard Bekenstein bound [1]

$$S \leqslant \frac{2\pi k_B}{\hbar c} RE = 2\pi RE, \tag{1}$$

where, as per the common practice in literature, we have set $\hbar = c = k_B = 1$. We will keep G explicit since we will discuss modification of the gravitational constant later. In general relativity, we can easily check that a Schwarzschild black hole saturates this bound. The Bekenstein bound also holds in non-gravitational system (not surprising — note the absence of G in the inequality), and is related to the positivity of the relative entropy [2,3]. The quantum informational perspective of the Bekenstein bound is still an active area of research [4]. The generality of the bound suggests that it could be fundamental, at least in asymptotically flat spacetimes.[1] On the other hand, in the literature, one finds many attempts to study what happens if we were to replace Bekenstein-Hawking entropy with various generalized entropies, such as Barrow entropy [6], Tsallis(-Cirto) entropy [7,8], Kaniadakis entropy [9–11], and Sharma-Mittal entropy [12], as well as generalizations thereof [13].

One of the motivations for contemplating these other possibilities is that the Bekenstein-Hawking entropy $S = A/4G = 4\pi G^2 M^2$ is not extensive in its energy M (that is, $S(\lambda M) \neq \lambda S(M)$), which means that its underlying statistics is probably not the Gibbs-Boltzmann entropy. As the logic goes, since the Bekenstein-Hawking entropy is not extensive anyway, one might as well consider other non-extensive forms that were first considered in non-gravitational systems, and use it to generalize the Bekenstein-Hawking form. (The Barrow case is somewhat special as it was motivated as a quantum gravity effect.) This certainly does not look like a strong motivation, and so we do not advocate for the applications of generalized entropy for black holes. While we need not believe in any of these extensions, it is important to study the effects of modifying Bekenstein-Hawking entropy on black hole physics. If the Bekenstein-Hawking entropy is in fact correct and therefore unique, then perhaps we can see what will go wrong by modifying it. (Similarly, modified gravity theories also often lead to subtle consistency issues.)

One thing that could potentially go *very* wrong is the Bekenstein bound. To see this, we consider for example, the Tsallis entropy, usually given in the context of black holes as[2]

$$S_T := \frac{A_0}{4G}\left(\frac{A}{A_0}\right)^{\delta} = \frac{A_0}{4G}\left(\frac{4\pi R^2}{A_0}\right)^{\delta}, \tag{2}$$

where A_0 is a constant of the theory. The right hand side (RHS) of the Bekenstein bound is

$$2\pi RE = 2\pi RM = \frac{\pi R^2}{G}. \tag{3}$$

The bound is only satisfied if

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[1] Its status in anti-de Sitter spacetimes are less clear. See [5].

[2] This may not be the case, see, e.g., [14,15].

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$$S_T^{1-\frac{1}{\delta}} \leqslant \left(\frac{A_0}{4G}\right)^{1-\frac{1}{\delta}} = \text{const.} \tag{4}$$

Evidently, since for fixed δ, the Tsallis entropy S_T is an increasing function of A, we see that the bound fails for *large* black holes. This was first pointed out in [16] (the example we use here is similar to their computation for the Barrow case). In general then, as argued in [16,17], when a generalized entropy is used in place of Bekenstein-Hawking one, we should not expect the Bekenstein bound to hold. Thus, if we take the position that the Bekenstein bound is fundamental, this would either imply that generalized entropies are not viable (and thus this can be used to constrain parameters of the theory; see e.g., [18]), or that we *cannot* naively directly apply any generalized entropy with the usual assumption that $R = 2GM$ and $E = M$. In fact, changing the entropy alone while keeping everything else unchanged has been long realized to be problematic and may be inconsistent (see, e.g., [19,20]; also see [21] for other criticisms in the context of cosmology), though this is still widely done in the literature.

In [22], we pointed out that if we were to take the claim that gravity is deeply related to thermodynamics seriously, then it is only natural that gravity will also need to be modified when we generalize the Bekenstein-Hawking entropy. We proposed that one possible way to do this is via generalizing Jacobson's method in deriving the Einstein field equations [23]. Specifically, we showed that if we generalized the Bekenstein Hawking entropy by changing the area A to some (dimensionful) function $f(A)$, namely

$$\frac{A}{4G} \longmapsto \frac{f(A)}{4G}, \tag{5}$$

then the resulting theory has a field equation that has the same form as general relativity, except that its gravitational constant G has to be replaced by

$$G \longmapsto G_{\text{eff}} := \frac{G}{f'(A)}, \tag{6}$$

where A is the area of the black hole horizon, and prime denotes derivative with respect to A. Since G_{eff} is in general a function unless the horizon is fixed, it is a novel class of varying-G theory. We shall refer to this approach as the GEVAG approach, short for "**G**eneralized **E**ntropy and **VA**rying-**G** gravity". In this work, because we only consider the Schwarzschild (static) solution, G_{eff} is fixed and we need not worry about its variation. Since the field equation of GEVAG has the same form as general relativity, the Schwarzschild solution has event horizon located at $r = 2G_{\text{eff}}M$. For more details, we refer the readers to [22].

This area dependence looks strange and suspicious, and we spent a great deal of effort making sense of it in [22]. Still, this claim has an unexpected supporting evidence. If we consider the usual logarithmic quantum gravity correction to the Bekenstein-Hawking entropy

$$S = \frac{A}{4G} + \tilde{c}\ln\left(\frac{A}{G}\right), \tag{7}$$

for some constant $\tilde{c}$, then applying the result above, we have

$$G_{\text{eff}} = \frac{G}{1 + \frac{\tilde{c}G}{A}}. \tag{8}$$

This is exactly the form one expects in the asymptotically safe gravity scenario:

$$G(k) = \frac{G(k_0)}{1 + \mathfrak{c}k^2}, \tag{9}$$

where k_0 is a reference energy scale and $\mathfrak{c}$ another constant. In fact, the authors in [24] (see also [25]) had argued that k is horizon area dependent: $k = \text{const.}/\sqrt{A}$. This implies Eq. (9) is exactly the same as Eq. (8). In other words, our GEVAG approach unexpectedly implies a connection between the standard logarithmic correction and the ASG approach. It is also interesting to note that the area dependence disappears when the entropy is linear in A only. Thus the Bekenstein-Hawking entropy is in a special class (unfortunately this does not fix the constant prefactor to be $1/4$).

Therefore, to check whether the Bekenstein bound is really consistent with generalized entropy, we need to check it in the GEVAG approach. In fact, based on the results already obtained in[3] [26], we have claimed in [22] that the Bekenstein bound is satisfied, possibly up to some constant prefactor. However, since Bekenstein bound wasn't the main focus therein, we did not elaborate on the details. In this work, we shall explicitly demonstrate this to be indeed the case with two explicit examples: Tsallis entropy and Rényi entropy. We then prove the general result for any generalized entropy $S = f(A)/4G$.

2. Bekenstein bound for Tsallis-Schwarzschild black hole

In the GEVAG approach, the gravitational constant for the Tsallis entropy case becomes, via Eq. (6),

$$G_{\text{eff}} = \frac{G}{\delta}\left(\frac{A}{A_0}\right)^{1-\delta}. \tag{10}$$

Furthermore, via the first law $dE = TdS$, where $T = 1/8\pi G_{\text{eff}}M$, one can show that the thermodynamic energy is not the same as the ADM mass M, but rather [22]

$$E = \frac{M}{2\delta - 1}. \tag{11}$$

As emphasized in [22], this relation is much simpler than what one would obtain from assuming $E \neq M$, but without the GEVAG consideration [26,27]. In addition, positivity of energies in Eq. (11) imposes the bound $\delta > 1/2$.

We now note that

$$S_T = \frac{A_0}{4G}\left(\frac{4\pi R^2}{A_0}\right)^{\delta} = \frac{A_0}{4G}\left[\frac{4\pi(2G_{\text{eff}}M)^2}{A_0}\right]^{\delta}. \tag{12}$$

On the other hand, Eq. (10) yields a relationship between G_{eff} and G:

$$G = \frac{M^{2(\delta-1)}(16\pi)^{\delta-1}A_0^{1-\delta}\delta}{G_{\text{eff}}^{1-2\delta}}. \tag{13}$$

We can substitute this into Eq. (12) and obtain

$$S = \frac{4\pi}{\delta}G_{\text{eff}}M^2. \tag{14}$$

On the other hand, the RHS of the Bekenstein bound is

$$2\pi RE = 2\pi(2G_{\text{eff}}M)\frac{M}{2\delta - 1} = \frac{4\pi}{2\delta - 1}G_{\text{eff}}M^2. \tag{15}$$

Therefore, $S/RE \leqslant 2\pi$ holds if and only if

$$\frac{1}{\delta} \leqslant \frac{1}{2\delta - 1} \Longleftrightarrow \delta \geqslant 2\delta - 1, \tag{16}$$

and thus for all $\delta \leqslant 1$. Otherwise, we can consider the "relaxed form" of Bekenstein bound by replacing the constant 2π with some constant C. Then the relaxed Bekenstein bound

$$S \leqslant CRE \tag{17}$$

is satisfied for S_T if we take the "Bekenstein constant" C to be

$$C = 2\pi\left(2 - \frac{1}{\delta}\right) \leqslant 4\pi. \tag{18}$$

Some clarifications are useful at this point. Let us first return to the original Bekenstein bound. The ratio S/RE is strictly less than 2π for non-black hole systems, and is saturated by a black hole. In principle, $S/RE \leqslant C$ for any $C \geqslant 2\pi$ would also be mathematically correct, but

[3] There are some problems and unclarified subtleties in [26], so the results in [22] superseded it.

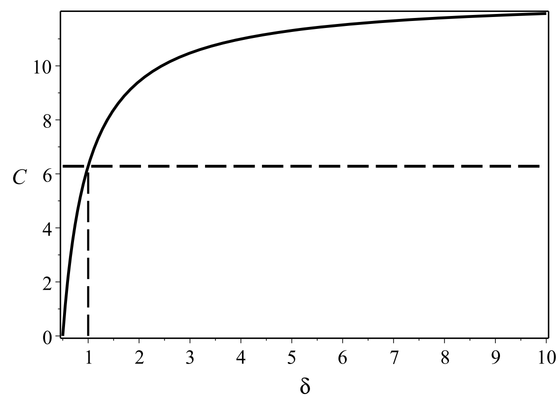

Fig. 1. The Bekenstein constant C for the Tsallis-Schwarzschild black hole is bounded between 0 and 4π, with $0 < C \leqslant 2\pi$ for $1/2 < \delta \leqslant 1$. The dashed vertical line and horizontal line are $\delta = 1$ and $C = 2\pi$, respectively.

physically $C > 2\pi$ would correspond to a hypothetical "hyper-entropic" object (i.e. an object whose entropy exceeds a black hole of the same energy). Such an object is likely to collapse into a black hole [28–31]. In fact, the Bekenstein bound means that such objects should be excluded — they would collapse into a black to restore the Bekenstein bound. For the same reason,[4] we *define* the "Bekenstein constant" C as the value that is saturated by a black hole in the case with generalized entropy.

Eq. (18) is an increasing function that reduces to 2π when $\delta \to 1$, consistent with the standard Bekenstein bound for Bekenstein-Hawking entropy. In fact, we can see from Fig. 1 that the original Bekenstein bound is satisfied for $\delta = 1$, and that C is strictly less than 2π, indicating an even tighter bound than the original version, provided that $1/2 < \delta \leqslant 1$. In the limit of large δ, we have $C \to 4\pi$, twice the standard value.

This suggests that the relaxed Bekenstein bound can also be classified as:

(1) Strong Bekenstein Bound: $0 < C < 2\pi$,
(2) (Original) Bekenstein Bound: $C = 2\pi$,
(3) Weak Bekenstein Bound: $2\pi < C$, with $C/2\pi = O(1)$.

Note that the strong bound is not necessarily good if C is too small. This is because one interpretation of the Bekenstein bound is that if a system has too much entropy it will collapse into a black hole, thus restoring the bound. If C is too small, this suggests black hole production is much easier, which will likely lead to conflicts with the observed number density of black hole of various masses.

3. Bekenstein bound for Rényi-Schwarzschild black hole

We now consider another form of entropy that has been widely considered in the literature: the Rényi entropy [32]. For black holes, this take the form [33,34]

$$S_R := \frac{1}{\alpha}\ln\left[1+\alpha\left(\frac{A}{4G}\right)\right],\ \alpha > 0, \tag{19}$$

for which the limit $\alpha \to 0$ recovers Bekenstein-Hawking entropy.

In the GEVAG approach, the gravitational constant for the Rényi case becomes, via Eq. (6),

$$G_{\text{eff}} = \frac{G}{f'(A)} = G\left(1+\frac{1}{4}\frac{\alpha A}{G}\right). \tag{20}$$

This can be solved in terms of G only by substituting in the Schwarzschild radius $r = 2G_{\text{eff}}M$. The result is

$$G_{\text{eff}} = \frac{1-\sqrt{1-16\pi\alpha G M^2}}{8\pi\alpha M^2}. \tag{21}$$

Thus the GEVAG approach readily implies a non-obvious upper bound for the Rényi parameter, which is crucial to ensure that $G_{\text{eff}} \in \mathbb{R}$:

$$\alpha \leqslant \frac{1}{16\pi G M^2}. \tag{22}$$

This bound was also found in [35], although with a different approach and assumptions.

Note that Eq. (20) implies

$$S_R := \frac{1}{\alpha}\ln\left[1+\alpha\left(\frac{A}{4G}\right)\right] = \frac{1}{\alpha}\ln\left(\frac{G_{\text{eff}}}{G}\right). \tag{23}$$

Let us now derive the thermodynamic energy. The first law is $dE = TdS_R$. The Hawking temperature of the black hole is $T = 1/8\pi G_{\text{eff}}M$ [22]. Substituting in the expression for S_R in Eq. (23), we can use the chain rule to get

$$dE = T\frac{dS_R}{dM}dM. \tag{24}$$

Since

$$\begin{aligned}\frac{dS_R}{dM} &= \frac{1}{\alpha}\frac{d}{dM}\left[\ln\left[1+\alpha\left(\frac{4\pi(2G_{\text{eff}}M)^2}{4G}\right)\right]\right] \\ &= \frac{1}{\alpha}\left(\frac{1-\sqrt{1-16\pi\alpha G M^2}}{M\sqrt{1-16\pi\alpha G M^2}}\right),\end{aligned} \tag{25}$$

upon simplifying the expression we finally obtain

$$dE = \frac{dM}{\sqrt{1-16\pi\alpha G M^2}}. \tag{26}$$

Integrating this finally gives the thermodynamic energy:

$$E = \frac{1}{4\sqrt{\pi\alpha G}}\arctan\left(\frac{4\sqrt{\pi\alpha G}M}{\sqrt{1-16\pi\alpha G M^2}}\right), \tag{27}$$

which is considerably more complicated than the Tsallis case. Let us define the dimensionless quantity

$$a := \alpha G M^2, \tag{28}$$

so that the energy-mass relation can be written as

$$E = M\left[\frac{1}{\sqrt{16\pi a}}\arctan\left(\frac{\sqrt{16\pi a}}{\sqrt{1-16\pi a}}\right)\right]. \tag{29}$$

We note that

$$\lim_{a\to 1/16\pi}\left[\arctan\left(\frac{\sqrt{16\pi a}}{\sqrt{1-16\pi a}}\right)\right] = \frac{\pi}{2}, \tag{30}$$

and

$$\lim_{a\to 1/16\pi}\frac{M}{\sqrt{16\pi a}} = M. \tag{31}$$

Therefore

$$\lim_{a\to 1/16\pi} E = \frac{\pi M}{2}. \tag{32}$$

Thus the ratio $E/M \to \pi/2$ in the limit $a \to 1/16$. The behavior of E/M is shown in Fig. 2. In the small a expansion, the thermodynamic energy E reduces to

$$E = M + \frac{8}{3}a\pi M + \cdots, \tag{33}$$

which smoothly reduces to $E = M$ in the limit $a \to 0$.

Again, we want to find the C in the relaxed Bekenstein bound $S \leqslant CRE$. Since C is defined to be the saturated value of the generalized Bekenstein bound, it can be obtained as $C = S/RE$. A direct computation yields

[4] Admittedly there is a caveat: whether hyper-entropic objects are indeed unstable for the case of generalized entropy requires more detailed investigations.

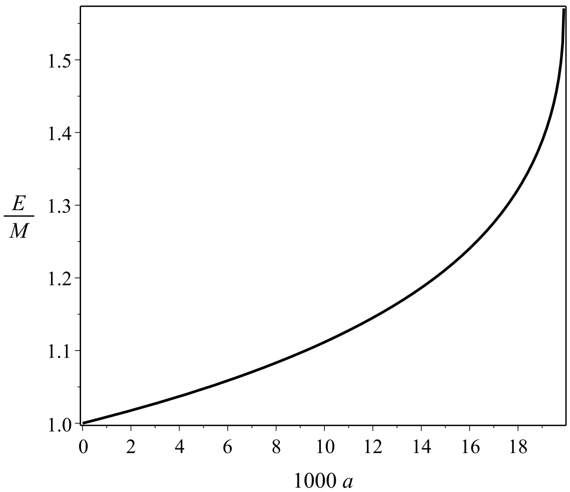

Fig. 2. The ratio E/M for the Rényi-Schwarzschild black hole. The horizontal axis has been re-scaled for clarity. The domain of a is $0 < a < 1/16\pi \approx 0.0199$. We have $E/M = 1$ when $a = 0$, the GR case. On the other hand, E/M tends to $\pi/2$ when $a \to 1/16\pi$.

$$C = \frac{\frac{1}{\alpha}\ln\left[1+\alpha\left(\frac{A}{4G}\right)\right]}{2G_{\text{eff}}M\left[\frac{1}{4\sqrt{\pi\alpha G}}\arctan\left(\frac{4\sqrt{\pi\alpha G}M}{\sqrt{1-16\pi\alpha GM^2}}\right)\right]}. \tag{34}$$

Substituting in $A = 4\pi(2G_{\text{eff}}M)^2$ and the expression for G_{eff} in Eq. (21), we finally obtain in terms of a,

$$C = \frac{16\sqrt{a\pi^3}\ln\left(\frac{1-\sqrt{1-16\pi a}}{8\pi a}\right)}{\left(1-\sqrt{1-16\pi a}\right)\arctan\left(\frac{4\sqrt{\pi a}}{\sqrt{1-16\pi a}}\right)}. \tag{35}$$

As shown in Fig. 3, unlike the Tsallis case, this is a decreasing function, so it always satisfies $C \leqslant 2\pi$. In this case the strong Bekenstein bound is strictly satisfied. In the limit $a \to 0$, we recover the standard value 2π for the Bekenstein constant. It should be emphasized that it is crucial to make the distinction between the strong Bekenstein bound and the standard Bekenstein bound. In the latter, black holes saturate the bound, while other systems satisfy strict inequality. Whereas for the former, *black holes* saturate a bound whose value C is less than 2π. Due to this physical difference, it would be misleading to say that such generalized entropy black holes satisfy the original bound.

4. Conclusion: general result

Given the vast literature of black holes with generalized entropy, it is crucial to clarify whether such black holes violate the Bekenstein bound or not. If one applies the generalized entropy directly, Bekenstein bound is indeed not expected to hold in general. However, instead of ruling out such generalized entropies, this could just be a hint that changing the entropy while keeping everything else unchanged is inconsistent. In this work, utilizing the GEVAG approach, we have explicitly demonstrated that both the Tsallis(-Cirto)-Schwarzschild black hole and the Rényi-Schwarzschild black hole satisfy the Bekenstein bound up to a constant term, as we claimed in [22].

Let us now present the general result for any generalized entropy $S = f(A)/4G$. For this to satisfy $S \leqslant CRE$, with $R = 2G_{\text{eff}}M$ for the Schwarzschild black hole under GEVAG scheme [22], it suffices to let the Bekenstein constant be

$$C = \frac{f(A)}{8GG_{\text{eff}}ME}. \tag{36}$$

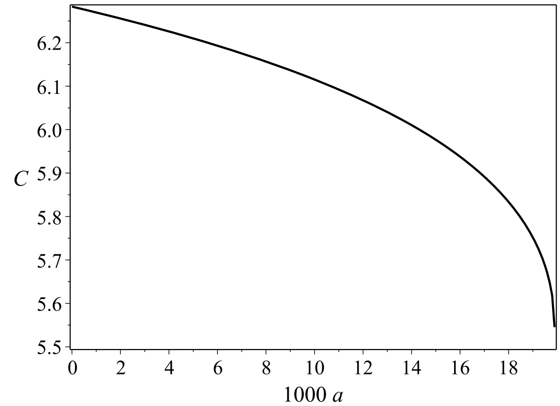

Fig. 3. The Bekenstein constant C for Rényi-Schwarzschild black hole, as a function of the dimensionless Rényi parameter $a := \alpha GM^2$. The horizontal axis has been re-scaled for clarity. The domain of a is $0 < a < 1/16\pi \approx 0.0199$. We have $C \to 2\pi$ in the limit $a \to 0$, and $C \to 8\ln 2 \approx 5.5452$ when $a \to 1/16\pi$.

This combination is clearly dimensionless and reduces to 2π in the GR limit. Furthermore, and crucially, we see that C is bounded because $f(A)$ should be bounded by assumption (otherwise the generalized entropy $S = f(A)/4G$ diverges, which does not make physical sense for a finite system). Likewise, $G_{\text{eff}} = G/f'(A)$ is nonzero as long as the entropy function $f(A)$ is differentiable and $f'(A)$ is finite. This is sensible for the same reason: if $f'(A)$ diverges, then f is becoming unbounded, which is not physical. The energy E should also be finite for any sensible thermodynamical system. Furthermore, since E should recover M in the GR limit, at least for small deviation away from the Bekenstein-Hawking entropy, E is guaranteed to be finite for any generalized entropy. (If for any unlikely reason, E is divergent but S finite, then the (weak) Bekenstein bound is trivially satisfied.)

In general, whether C defined by Eq. (36) is necessarily less than 2π has to be checked on a case by case basis. In our opinion, even if the prefactor is not 2π but any other finite number larger than 2π, this should not be considered as a true violation of the Bekenstein bound (cf. the gross violation in Sec. 1), since its essential physics that the entropy should be bounded above by the product RE remains intact. Of course, if C is too large, one may start to question whether we want to accept such a theory, but if $C > 2\pi$ but $C/2\pi$ is only $O(1)$ or even $O(10)$, such a "weak Bekenstein bound" is arguably still reasonable.

We therefore conclude that the Bekenstein bound, at least a "relaxed" form, remains valid if we consistently apply generalized entropy, namely by taking into account the effects of generalized entropy on the theory of gravity itself, via the GEVAG scheme. This sensible results also, in turn, lend credence to the GEVAG approach. Of course, if one insists on requiring that the deviation of C from 2π is not too large, then one can still use it to constrain the parameter of the theory.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

References

[1] Jacob D. Bekenstein, Universal upper bound on the entropy-to-energy ratio for bounded systems, Phys. Rev. D 23 (1981) 287.
[2] Horacio Casini, Relative entropy and the Bekenstein bound, Class. Quantum Gravity 25 (2008) 205021.

[3] Roberto Longo, A Bekenstein-type bound in QFT, Commun. Math. Phys. 406 (2025) 95, arXiv:2409.14408 [math-ph].
[4] Patrick Hayden, Jinzhao Wang, What exactly does Bekenstein bound?, Quantum 9 (2025) 1664, arXiv:2309.07436 [hep-th].
[5] Gary W. Gibbons, Malcolm J. Perry, Christopher N. Pope, Partition functions, the Bekenstein bound and temperature inversion in anti-de Sitter space and its conformal boundary, Phys. Rev. D 74 (2006) 084009, arXiv:hep-th/0606186 [hep-th].
[6] John D. Barrow, The area of a rough black hole, Phys. Lett. B 808 (2020) 135643, arXiv:2004.09444 [gr-qc].
[7] Constantino Tsallis, Possible generalization of Boltzmann-Gibbs statistics, J. Stat. Phys. 52 (1988) 479.
[8] Constantino Tsallis, Leonardo J.L. Cirto, Black hole thermodynamical entropy, Eur. Phys. J. C 73 (2013) 2487, arXiv:1202.2154 [cond-mat.stat-mech].
[9] Giorgio Kaniadakis, Statistical mechanics in the context of special relativity, Phys. Rev. E 66 (2022) 056125, arXiv:cond-mat/0210467 [cond-mat.stat-mech].
[10] Giorgio Kaniadakis, Statistical mechanics in the context of special relativity II, Phys. Rev. E 72 (2005) 036108, arXiv:cond-mat/0507311 [cond-mat.stat-mech].
[11] Niki Drepanou, Andreas Lymperis, Emmanuel N. Saridakis, Kuralay Yesmakhanova, Kaniadakis holographic dark energy and cosmology, Eur. Phys. J. C 82 (2022) 449, arXiv:2109.09181 [gr-qc].
[12] A. Sayahian Jahromi, S.A. Moosavi, H. Moradpour, J.P. Morais Graça, I.P. Lobo, I.G. Salako, A. Jawad, Generalized entropy formalism and a new holographic dark energy model, Phys. Lett. B 780 (2018) 21, arXiv:1802.07722 [gr-qc].
[13] Shin'ichi Nojiri, Sergei D. Odintsov, Valerio Faraoni, How fundamental is entropy? From non-extensive statistics and black hole physics to the holographic dark universe, Phys. Rev. D 105 (2022) 044042, arXiv:2201.02424 [gr-qc].
[14] Phuwadon Chunaksorn, Ratchaphat Nakarachinda, Pitayuth Wongjun, Black hole thermodynamics via Tsallis statistical mechanics, Eur. Phys. J. C 85 (2025) 532, arXiv:2502.02522 [gr-qc].
[15] Everton M.C. Abreu, Jorge Ananias Neto, From modified Tsallis-Renyi entropy to a Mond-like force law, Bekenstein bound, and Landauer principle for black holes, Phys. Lett. B 866 (2025) 139579, arXiv:2505.03061 [gr-qc].
[16] Everton M.C. Abreu, Jorge Ananias Neto, Statistical approaches and the Bekenstein bound conjecture in Schwarzschild black holes, Phys. Lett. B 835 (2022) 137565, arXiv:2207.13652 [gr-qc].
[17] Mehdi Shokri, Bekenstein bound on black hole entropy in non-Gaussian statistics, Phys. Lett. B 860 (2025) 139193, arXiv:2411.00694 [hep-th].
[18] Gabriella V. Ambrósio, Michelly S. Andrade, Paulo R.F. Alves, Cleber N. Costa, Jorge Ananias Neto, Ronaldo Thibes, Exploring modified Kaniadakis entropy: MOND theory and the Bekenstein bound conjecture, arXiv:2405.14799 [gr-qc].
[19] Shin'ichi Nojiri, Sergei D. Odintsov, Valerio Faraoni, Alternative entropies and consistent black hole thermodynamics, Int. J. Geom. Methods Mod. Phys. 19 (2022) 2250210, arXiv:2207.07905 [gr-qc].
[20] Shin'ichi Nojiri, Sergei D. Odintsov, Valerio Faraoni, Area-law versus Rényi and Tsallis black hole entropies, Phys. Rev. D 104 (2021) 084030, arXiv:2109.05315 [gr-qc].
[21] Hussain Gohar, Vincenzo Salzano, On the foundations of entropic cosmologies: inconsistencies, possible solutions and dead end signs, Phys. Lett. B 855 (2024) 138781, arXiv:2307.01768 [gr-qc].
[22] Hengxin Lü, Sofia Di Gennaro, Yen Chin Ong, Generalized entropy implies varying-G: horizon area dependent field equations and black hole-cosmology coupling, Ann. Phys. 474 (2025) 169914, arXiv:2407.00484 [gr-qc].
[23] Ted Jacobson, Thermodynamics of spacetime: the Einstein equation of state, Phys. Rev. Lett. 75 (1995) 1260, arXiv:gr-qc/9504004.
[24] Chiang-Mei Chen, Yi Chen, Akihiro Ishibashi, Nobuyoshi Ohta, Daiki Yamaguchi, Running Newton coupling, scale identification and black hole thermodynamics, Phys. Rev. D 105 (2022) 106026, arXiv:2204.09892 [hep-th].
[25] Chiang-Mei Chen, Yi Chen, Akihiro Ishibashi, Nobuyoshi Ohta, Quantum improved regular Kerr black holes, Chin. J. Phys. 92 (2024) 766, arXiv:2308.16356 [hep-th].
[26] Sofia Di Gennaro, Hao Xu, Yen Chin Ong, How Barrow entropy modifies gravity: with comments on Tsallis entropy, Eur. Phys. J. C 82 (2022) 1066, arXiv:2207.09271 [gr-qc].
[27] H. Moradpour, A.H. Ziaie, Iarley P. Lobo, J.P. Morais Graça, U.K. Sharma, A. Sayahian Jahromi, The third law of thermodynamics, non-extensivity, and energy definition in black hole physics, Mod. Phys. Lett. A 37 (2022) 2250076, arXiv:2106.00378 [gr-qc].
[28] Stephen D.H. Hsu, David Reeb, Monsters, black holes and the statistical mechanics of gravity, Mod. Phys. Lett. A 24 (2009) 1875, arXiv:0908.1265 [gr-qc].
[29] Stephen D.H. Hsu, David Reeb, Black hole entropy, curved space and monsters, Phys. Lett. B 658 (2008) 244, arXiv:0706.3239 [hep-th].
[30] Yen Chin Ong, Pisin Chen, The fate of monsters in anti-de Sitter spacetime, J. High Energy Phys. 07 (2013) 147, arXiv:1304.3803 [hep-th].
[31] Raphael Bousso, Arvin Shahbazi-Moghaddam, Singularities from entropy, Phys. Rev. Lett. 128 (2022) 231301, arXiv:2201.11132 [hep-th].
[32] Alfréd Rényi, On measures of information and entropy, in: Proceedings of the Fourth Berkeley Symposium on Mathematics, Statistics and Probability, 1961, pp. 547–561.
[33] Tamás S. Biró, Viktor G. Czinner, A q-parameter bound for particle spectra based on black hole thermodynamics with Rényi-entropy, Phys. Lett. B 726 (2013) 861, arXiv:1309.4261 [gr-qc].
[34] Viktor G. Czinner, Hideo Iguchi, Rényi entropy and the thermodynamic stability of black holes, Phys. Lett. B 752 (2016) 306, arXiv:1511.06963 [gr-qc].
[35] Viktor G. Czinner, Hideo Iguchi, Hawking-Rényi black hole thermodynamics, Kiselev solution, and cosmic censorship, Eur. Phys. J. C 85 (2025) 443, arXiv:2504.16705 [gr-qc].